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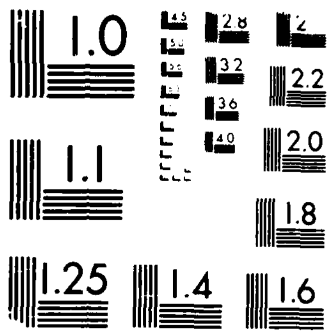

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APPLICATIONS OF SIGNAL PROCESSING

IN DIGITAL COMMUNICATIONS

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We consider the design of multidimensional signal sets and their combination with block or trellis codes. The goal is to achieve a high efficiency in the use of frequency spectrum for digital communications.

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MULTIDIMENSIONAL MODULATION AND CODING FOR BANDLIMITED DIGITAL CHANNELS

by

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ABSTRACT

A class of multidimensional signals, based on what we call Generalized Group Alphabets, is introduced, and its basic properties are derived. The combination of Generalized Group Alphabets and coding is also examined: two coding schemes are considered, viz., Ungerboeck's scheme for combination with convolutional codes, and Ginzburg's scheme for combination with block codes. The performance of these schemes makes them attractive for transmission over bandlimited digital channels.

This work has been sponsored in part by the United States Army through its European Research Office, and in part by the Italian Department of Education under a "60% Grant".

Our research activity during the period covered by this report was devoted to the design of multidimensional codes for bandlimited digital channels. The goal is to achieve a high efficiency in the use of the available spectrum through trellis or block codes that operate in a multidimensional signal space.

In particular, our attention was focused on the algebraic properties of a class of multidimensional signal sets, which we call "Generalized Group Alphabets". Further details about this project can be found in the enclosed manuscript, which was submitted for publication in the IEEE Transactions on Information Theory.

family. After a description of the main features of these alphabets, we show how they can be used in conjunction with error control codes; the concept of fair partition will be introduced, and some of its relevant properties described. Finally, we provide some examples of actual designs.

II. GENERALIZED GROUP ALPHABETS

Consider a set of K n-vectors $\underline{X} = \{X_1,\ldots,X_K\}$, called the initial set, and L orthogonal n x n matrices $S_1,\ldots, S_L$ that form a finite group $\underline{G}$ under multiplication.

Definition 1

The set of vectors $\underline{G}X_1$, $\underline{G}X_2$, ... , $\underline{G}X_K$ obtained from the action of $\underline{G}$ on the vectors of the initial set is called a Generalized Group Alphabet (GGA). $\underline{G}$ is called its generating group.

Definition 2

A GGA is called separable if the vectors of the initial set are transformed by $\underline{G}$ into either disjoint or coincident vector sets, i.e.,

$$\underline{G}X_j \cap \underline{G}X_k = \begin{cases} \emptyset & j \neq k \\ \underline{G}X_j & j = k \end{cases}$$

If $\|X\|$ denotes the Euclidean length of a vector X, the quantity $\|X\|^2$ is proportional to the energy of the signal associated with X for transmission over a continuous channel. Since an orthogonal matrix transforms a vector into one with the same length, the signals associated with a GGA have as many energy levels as there are in the initial set. The special case of a GGA with K=1, and hence only one energy level, was extensively studied in [7].

Definition 3

A GGA is called regular if the number of vectors in each subalphabet $\underline{G}X_j$, j=1,...,K, does not depend on j, i.e., each vector of the initial set is transformed by $\underline{G}$ into the same number of distinct vectors. A regular GGA is called strongly regular if each set $\underline{G}X_j$ contains exactly L distinct vectors.

The following result stems directly from the definitions.

Proposition 1

The number M of vectors in a regular GGA is a multiple of K. If GGA is strongly regular, then M=KL.

Hereafter we exhibit four examples of these alphabets. Notice that for K=1 every GGA is regular, but not necessarily strongly regular [7,16].

Alphabet 1 (Asymmetric M-PSK: 2 dimensions, 1 energy level)

Choose an initial vector $X = (\cos\theta, \sin\theta)$, θ a given constant, an integer $M=2^{\mu}$, and consider the group of 2 x 2 orthogonal matrices of the form R^iT^j, $i=0,1, \ldots, M-1$, $j=1,2$, where

$$R = \begin{bmatrix} \cos(2\pi/M) & \sin(2\pi/M) \\ -\sin(2\pi/M) & \cos(2\pi/M) \end{bmatrix}$$

and

$$T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

It is seen that the effect of R on a 2-dimensional vector is to rotate it by an angle $2\pi/M$, and the effect of T is to exchange its components. This group has 2M elements, and gives rise to a separable alphabet of M or 2M vectors, according to the choice of the initial vector. Notice that the alphabet is strongly regular only when it has 2M elements (asymmetric M-PSK [13,14]).

Alphabet 2 (4 dimensions, 1 energy level)

Consider the group of matrices which act on a four-dimensional initial vector by permuting its components and replacing them with their negatives. This group has $4!2^4$ elements. If the initial vector is $X_1=(a,a,a,0)$, $a=1/\sqrt{3}$, the resulting (separable) alphabet has M=32 distinct unit-energy vectors (see Fig.1).

<u>Alphabet 3</u> (2 dimensions, 3 energy levels)
Our third example is shown in Fig.2. Points 1,2,3 and 4 denote the four vectors in the initial set. The matrices generating the code are those associated to plane rotations by multiples of $\pi/2$. The resulting (strongly regular, separable) alphabet is the conventional 16-QAM.

<u>Alphabet 4</u> (4 dimensions, 2 energy levels)
This alphabet which has two energy levels, K=4, and M=128, is obtained from the initial set of vectors

$$\begin{matrix} c & c & c & 0 \\ -b & c & c & 0 \\ c & -b & c & 0 \\ c & c & -b & 0 \end{matrix}$$

with c=0.389 and b=0.939. If we apply to this initial set the same matrix group which generates Alphabet 2, we get a separable alphabet with 128 vectors (see Fig.3). Among them, 32 have energy $3c^2$, and 96 have energy b^2+2c^2. The average energy is 1.

We consider now some distance properties of the elements of a GGA. Choose a partition of it into m subsets $\underline{Z}_1, \underline{Z}_2, \ldots, \underline{Z}_m$. For each subset $\underline{Z}_i$, we can define the <u>intradistance set</u> as the set of all the Euclidean distances among pairs of vectors in $\underline{Z}_i$. For any pair of distinct subsets $\underline{Z}_i$, $\underline{Z}_j$, we define their <u>interdistance set</u> as the set of all the Euclidean distances between a vector in $\underline{Z}_i$ and a vector in $\underline{Z}_j$.

<u>Definition 4</u>
The partition of a separable GGA into m subsets $\underline{Z}_1,\ldots,\underline{Z}_m$ is called <u>fair</u> if all the subsets are distinct, include the same

number of vectors, and their intradistance sets are equal.

We shall now exhibit a constructive method to generate fair partitions of a GGA. Consider the generating group $\underline{G}$ of the GGA, one of its subgroups, say $\underline{H}$, and the partition of $\underline{G}$ into left cosets of $\underline{H}$. We have the following result.

Theorem 1

If the left cosets of the subgroup $\underline{H}$ are applied to the initial set of a strongly regular GGA, this procedure results into a fair partition of the GGA. Under the same hypotheses, if $\underline{H}$ is a normal subgroup, then left and right cosets give rise to the same fair partition.

Proof

Let S denote an element of $\underline{G}_g$ not belonging to $\underline{H}$, and $S\underline{H}$ the corresponding left coset. If X_i, X_j are two (not necessarily distinct) vectors of the initial set, and S_h, S_k are two elements of $\underline{H}$, the intradistance set associated with the coset $S\underline{H}$ includes the quantities

$$d^2_{ij}(S, S_h, S_k) \triangleq \| S\, S_h\, X_j - S\, S_k\, X_i \|^2$$

as S_h, S_k run in $\underline{H}$, and X_i, X_j run in the initial vector set. We have

$$d^2_{ij}(S, S_h, S_k) = \|X_j\|^2 + \|X_i\|^2 - 2\, X_j^T\, S_h^T S^T\, S\, S_k\, X_i =$$

$$= \|X_j\|^2 + \|X_i\|^2 - 2\, X_j^T\, S_h^T\, S_k\, X_i$$

where the superscript T denotes transpose.

As the right hand side of the last equation does not depend on S, we have shown that the intradistance set associated with the left cosets of $\underline{H}$ are independent of the coset. Moreover, if $\underline{H}$ is normal, then right cosets and left cosets give rise to the the same fair partition: in fact, normality implies that for every S

$$S\underline{H}=\underline{H}S. \blacksquare$$

The condition of strong regularity of the GGA can be removed: but in this case it may happen that different cosets generate the same element of the partition. Hence, some of the cosets must be removed from consideration. Moreover, notice that if $\underline{H}$ is a normal subgroup of $\underline{G}$, then we do not need to distinguish between left or right coset partitions. On the contrary, if $\underline{H}$ is not normal, the partitions obtained from right cosets may not be fair, as shown by the following counterexample.

Example 1

Let us consider the 4-dimensional alphabet generated by the action of the natural matrix representation of the permutation group $\underline{S}_4$ on the initial vector (-3d/2, - d/2, d/2, 3d/2), d a constant. Let us consider the partition induced by the subgroup $\underline{H}$ of the matrices leaving invariant the fourth component of the initial vector. This subgroup is isomorphic to $\underline{S}_3$. The left and right coset partitions associated with $\underline{H}$ are shown in Table I. It can be seen that the partition associated with right cosets is not fair, because its intradistance sets are not equal.

In some cases, we are interested to partition further every element $\underline{Z}_i$ in the same number of subsets. We are lead to the concept of a chain partition.

Definition 5

The chain partition of a separable GGA is called fair if any two elements of the partition at the same level of the chain include the same number of vectors and have equal intradistance sets.

left coset partition	right coset partition
(-3d/2,- d/2, d/2, 3d/2)	(-3d/2,- d/2, d/2, 3d/2)
(- d/2,-3d/2, d/2, 3d/2)	(- d/2,-3d/2, d/2, 3d/2)
(d/2,- d/2,-3d/2, 3d/2)	(d/2,- d/2,-3d/2, 3d/2)
(-3d/2, d/2,- d/2, 3d/2)	(-3d/2, d/2,- d/2, 3d/2)
(- d/2, d/2,-3d/2, 3d/2)	(- d/2, d/2,-3d/2, 3d/2)
(d/2,-3d/2,- d/2, 3d/2)	(d/2,-3d/2,- d/2, 3d/2)
(3d/2,-3d/2,- d/2, d/2)	(3d/2,- d/2, d/2,-3d/2)
(3d/2,- d/2,-3d/2, d/2)	(- d/2, 3d/2, d/2,-3d/2)
(3d/2, d/2,- d/2,-3d/2)	(d/2,- d/2, 3d/2,-3d/2)
(3d/2,-3d/2, d/2,- d/2)	(3d/2, d/2,- d/2,-3d/2)
(3d/2,- d/2, d/2,-3d/2)	(- d/2, d/2, 3d/2,-3d/2)
(3d/2, d/2,-3d/2,- d/2)	(d/2, 3d/2,- d/2,-3d/2)
(-3d/2, 3d/2,- d/2, d/2)	(-3d/2, 3d/2, d/2,- d/2)
(- d/2, 3d/2,-3d/2, d/2)	(- d/2,-3d/2, d/2,- d/2)
(d/2, 3d/2,- d/2,-3d/2)	(d/2, 3d/2,-3d/2,- d/2)
(-3d/2, 3d/2, d/2,- d/2)	(-3d/2, d/2, 3d/2,- d/2)
(- d/2, 3d/2, d/2,-3d/2)	(3d/2, d/2,-3d/2,- d/2)
(d/2, 3d/2,-3d/2,- d/2)	(d/2,-3d/2, 3d/2,- d/2)
(-3d/2,- d/2, 3d/2, d/2)	(-3d/2,- d/2, 3d/2, d/2)
(- d/2,-3d/2, 3d/2, d/2)	(- d/2,-3d/2, 3d/2, d/2)
(d/2,- d/2, 3d/2,-3d/2)	(3d/2,- d/2,-3d/2, d/2)
(-3d/2, d/2, 3d/2,- d/2)	(-3d/2, 3d/2,- d/2, d/2)
(- d/2, d/2, 3d/2,-3d/2)	(- d/2, 3d/2,-3d/2, d/2)
(d/2,-3d/2, 3d/2,- d/2)	(3d/2,-3d/2,- d/2, d/2)

Table I - Left and Right coset partitions of GGA

For fair chain partitions we have the following theorem, whose proof is straighforward and will be omitted.

Theorem 2

Consider a strongly regular GGA, and a chain of subgroups of its generating group $\underline{G}$, that is

$$\underline{H}_1 \subset \underline{H}_2 \subset \underline{H}_3 \subset \ldots \subset \underline{H}_s = \underline{G} .$$

Use $\underline{H}_{s-1}$ and its left cosets to generate a partition of GGA. Then, use $\underline{H}_{s-1}$ and its left cosets in $\underline{H}_s$ to further partition all the sets of the previous partition. Repeat the procedure with $\underline{H}_{s-2}$, and so on, until $\underline{H}_1$ and its left cosets in $\underline{H}_2$ are used. The resulting chain partition of GGA is fair.

A theorem concerning the interdistance sets sheds some further light on the symmetry properties of GGA's.

Theorem 3

Let $\underline{H}$ be a normal subgroup of $\underline{G}$. The partition of a strongly regular GGA obtained by applying the left cosets of $\underline{H}$ to the initial set $\underline{X}$ has the following property: The interdistance set associated with any two cosets, say $S_1\underline{H}$ and $S_2\underline{H}$, is a function only of the coset $S_3\underline{H}$, where $S_3 = S_1^T S_2$, and not of S_1, S_2 separately.

Proof

Let S_1 and S_2 denote two coset leaders. If X_i, X_j are two (not necessarily distinct) vectors of the initial set $\underline{X}$, and S_h, S_k are two elements of $\underline{H}$, the distances among elements of the cosets $S_1\underline{H}$ and $S_2\underline{H}$ includes the quantities

$$d_{ij}(S_1, S_2, S_h, S_k) \triangleq \| S_1 S_h X_j - S_2 S_k X_i \|$$

as S_h, S_k run in $\underline{H}$ and X_i, X_j run in $\underline{X}$. We have

$$d_{ij}^2(S_1, S_2, S_h, S_k) = \|X_j\|^2 + \|X_i\|^2 - 2\, X_j^T\, S_h^T S_1^T\, S_2 S_k\, X_i =$$

$$= \|X_j\|^2 + \|X_i\|^2 \;\; - 2\, X_j^T\, S_h^T\, S_3\, S_k X_i$$

Finally, as $\underline{H}$ is a normal subgroup, we have

$$S_1\underline{H}\; S_2\underline{H} = S_1 S_2 \underline{H} = S_3\underline{H}$$

i.e., $S_3\underline{H}$ is another coset. ■

We now provide some examples of fair partitions of a GGA. Consider first the rotation group which generates Alphabet 3 (see Fig.2) and its partition into the two cosets associated with the rotations 0, π, and $\pi/2$, $-\pi/2$, respectively. The GGA is fairly partitioned into the two subalphabets {1,2,3,4,9,10,11,12} and {5,6,7,8,13,14,15,16}.

Fig.1 shows a fair partition of the Alphabet 2 in four subsets of 8 vectors each. This partition is obtained as follows: denote by α the orthogonal matrix whose effect on a vector is to cyclically shift its components to the right by one position, and to change sign to the second component. Then the set

$$\underline{H} = \{\alpha^0,\alpha^1,\alpha^2,\alpha^3,\alpha^4,\alpha^5,\alpha^6,\alpha^7\}$$

is a cyclic normal subgroup of the group $\underline{G}$ generating the alphabet, and its cosets generate the fair partition.

A fair partition of Alphabet 4 into 16 subsets of 8 vectors each stems from the subgroup {I, -I}, where I is the 4 x 4 identity matrix (see Fig.3). A fair partition of Alphabet 1 is obtained by considering the two cosets of the subgroup $\{R_i\}_{i=0}^{M-1}$.

Definition 6

Let $\underline{R}$ be a left coset of $\underline{G}$ in the fair partition of a GGA, and S_g an element of $\underline{G}$. We define the distance profile [15] associated with $\underline{R}$ and S_g as the polynomial in the indeterminate w:

$$F(w, S_g, R) \triangleq \sum_{d^2} a(d^2)\, w^{d^2}$$

where $a(d^2)$ is the number of elements of $\underline{RX}$ that have squared distance d^2 with respect to an element of the set $S_g\underline{RX}$.

Example 2

Consider $K=1$, $X_1=(1, 0)^T$, and the group of plane rotations

$$S_i = \begin{bmatrix} \cos(i\pi/2) & \sin(i\pi/2) \\ -\sin(i\pi/2) & \cos(i\pi/2) \end{bmatrix}, \quad i=0,1,2,3$$

The subgroup $\{S_0, S_2\}$ is normal. The distance profiles are summarized in Table II.

$\underline{R}$	S_g	$F(w, S_g, \underline{R})$
$\{S_0, S_2\}$	S_0	$2w^0+2w^4$
$\{S_0, S_2\}$	S_1	$4w^2$
$\{S_0, S_2\}$	S_2	$2w^0+2w^4$
$\{S_0, S_2\}$	S_3	$4w^2$
$\{S_1, S_3\}$	S_0	$2w^0+2w^4$
$\{S_1, S_3\}$	S_1	$4w^2$
$\{S_1, S_3\}$	S_2	$2w^0+2w^4$
$\{S_1, S_3\}$	S_3	$4w^2$

Table II - Distance profiles for Example 2

Definition 7

A fair partition of a GGA is called homogeneous if the set $\{F(w,S,\underline{R})\}_{S\varepsilon\underline{G}}$ does not depend on $\underline{R}$. It is called strongly homogeneous if $F(w,S,\underline{R})$ does not depend on $\underline{R}$ for any S.

Theorem 4

If $\underline{G}$ is a commutative group, all the partitions generated by its subgroups are fair and strongly homogeneous.

Proof

Let $\underline{H}$ be a subgroup of $\underline{G}$: this is obviously normal, so that the partition induced by $\underline{H}$ is fair. Let X_i, X_j be two elements of the initial set $\underline{X}$, S an element of $\underline{G}$, S_H an element of $\underline{H}$. Then for any $S_g \varepsilon \underline{G}$ the computation of $F(w, S_g, S\underline{H})$ involves enumerating the squared distances

$$\| S\, S_H\, X_i - S_g S\, S_{1H} X_j \|^2 = \| S\, S_H\, X_i - S\, S_g S_{1H} X_j \|^2 =$$

$$= \| S_H\, X_i - S_g S_{1H} X_j \|^2$$

which do not depend on S, and hence on the element of the fair partition. ∎

Theorem 5

If $\underline{H}$ is a subgroup of $\underline{G}$ in a strongly regular GGA, the partition generated by the left cosets of $\underline{H}$ is fair and homogeneous.

Proof

Let $\underline{H}$ be a subgroup of $\underline{G}$. Then, the partition induced by the left cosets of $\underline{H}$ is fair. Let X_i, X_j be two elements of the initial set, S an element of $\underline{G}$, S_H and S_{1H} two elements of $\underline{H}$. Then for any $S_g \epsilon G$ the computation of $F(w, S_g, S\underline{H})$ involves enumerating the squared distances

$$\| S\, S_H\, X_i - S_g S\, S_{1H} X_j \|^2 = \| S_H\, X_i - S^T\, S_g S\, S_{1H} X_j \|^2 =$$

$$= \| S_H\, X_i - S_g' S_{1H} X_j \|^2$$

so that $F(w,S_g,S\underline{H}) = F(w,S'_g,S\underline{H})$, and as S_g runs through $\underline{G}$ all over G also $S' = S_g^T\, S\, S$ does. Thus, the assertion is proved. ∎

III. MULTIDIMENSIONAL CODED SIGNALS. BLOCK CODES

We shall now see how the multidimensional alphabets described in the previous section can be used in conjunction with codes to further enhance their performance. In this section, we shall focus our attention on block codes, while next section will be devoted to convolutional (trellis) codes.

Hinai and Hirakawa [18] and V.V. Ginzburg [8] have recently described constructions which make it possible to design alphabets with an arbitrary signal distance and with a regular structure, as insured by the algebraic properties of block codes. Fig.4 shows Ginzburg's construction. The L block encoders $C_1, C_2, \ldots, C_L$ accept source symbols, and output L blocks $(q_{1j}, q_{2j}, \ldots, q_{Nj})$ of N symbols each. The modulator f maps each L-tuple $(q_{j1}, \ldots, q_{jL})$, $j=1, \ldots, N$, into the vector

$$x_j = f(q_{j1}, \ldots, q_{jL}), \quad j = 1, \ldots, N$$

chosen from a set $\underline{A}$ of $M=M_1 \ldots M_L$ elements. This mapping is obtained as follows. In the set $\underline{A}$ we define a system of L partitions such that each class of the ℓ-th partition includes M_ℓ classes of the $(\ell-1)$-th partition, so that it will consist of $M(\ell)=M_1 M_2 \ldots M_\ell$ signals. By numbering the classes of the $(\ell-1)$-th level occurring in a class of the ℓ-th level we can obtain a one-to-one mapping of the set of classes of the $(\ell-1)$-th partition onto the set of integers $\{0, \ldots, M_\ell-1\}$. Therefore, if q_{ij} are chosen in the set $\{0, \ldots, M_\ell-1\}$, $\ell=1, \ldots, L$, any L-tuple $(q_{j1}, \ldots, q_{jL})$ defines a unique value of the j-th elementary signal $x_j = f(q_{j1}, \ldots, q_{jL})$ (see Figure 5).

We have the following result [8]: the alphabet obtained has a minimum squared Euclidean distance D^2 such that

$$D^2 \geq \min_{1 \leq \ell \leq L} (\delta_\ell^2 d_\ell)$$

where $d_1, \ldots, d_L$ are the minimum Hamming distances of the L block codes $C_1, \ldots, C_L$, and $\delta^2{}_i$ is the minumum squared Euclidean distance between the symbols in each subalphabet of the i-th partition.

Consider now Ginzburg's constructions based on generalized group alphabets. By associating with each level the elements of a fair partition (the concept of a fair chain can be used here), all the subalphabets at a given level have the same minimum distance. From the fair partition of Alphabet 2 described before, we have $\delta^2{}_1=2/3$, and $\delta^2{}_2=2$. Thus, using the (N,k,3) Hamming code on GF(4) [9 ,p.193-4] and the trivial (N,N,1) code on GF(8), with $N=(4^m-1)/3$, $k=N-m$, $m\geq 1$, we have $D^2 \geq 2$. The resulting alphabet has a rate

$$R = [\ 5(4^m - 1) - 6\ m\ ]/[\ 4\ (4^m - 1)]$$

and

$$D^2 \log_2 M = 10 - 12\ m\ /\ (\ 4^m - 1\ )$$

For example, choosing $m=2$ we get a rate $R=1.05$ and $D^2\log_2 M \geq 8.4$; with $m=3$ we get $R=1.18$ and $D^2\log_2 M \geq 9.4$.

Using Alphabet 4, and the partition described, we have $\delta^2{}_1=2c^2$, and $\delta^2{}_2=8c^2$. The (18,15,4) extended Hamming code [19,p.36] on GF(16) and the trivial (18,18,1) code on GF(8) can be employed, providing a squared minimum distance $D^2 \geq 1.211$. This alphabet yields $R=1.583$ and $D^2\log_2 M \geq 7.67$.

IV. MULTIDIMENSIONAL CODED SIGNALS. TRELLIS (UNGERBOECK) CODES

We shall now see how an Ungerboeck codes [10] can be designed using a multidimensional alphabet generated as described in Section II. Such codes can be specified as in [17]. Each coded symbol depends on $k+\nu$ source bits, namely the block $\tau=(a_1,\ldots,a_k)$ of k bits generated by the source, plus ν bits preceding this block. The ν bits determine one of the $N=2^{\nu}$ states of the encoder, say $\sigma = (a_{k+1}, \ldots, a_{k+\nu})$, $a_n=0,1$. The encoder state for the next coded symbol is obtained by shifting the a_n's k places to the right, dropping the right-most k bits and inserting on the left the most recent k source bits. The encoded symbol x depends on τ and σ; we write

$$x = f(\tau, \sigma) \tag{4.1}$$

where x is an element of a GGA. This encoding procedure can be described using a trellis (Fig.6 shows a section of such a trellis, obtained for $\nu=2$).

Although no formal proof exists, it is conjectured that a good code should show a good deal of symmetry, to be reflected by the structure of the function f in (4.1), or, equivalently, by the assignment of symbols to the branches connecting any pair of nodes in the code trellis (for further details see, e.g., [10], [11]). This can be obtained in our framework by assigning to the branches associated with each node the set of symbols obtained from a fair partition of a GGA. This is equivalent to the procedure suggested in [10] and called "mapping by set partitioning": thus, our procedure can be viewed as a systematic way to achieve set partitioning.

The most widely used single parameter that specifies the performance of these codes on the additive white Gaussian noise channel is the <u>free distance</u>. This can be computed using a generating function approach, which consists of enumerating all possible distances between sequences of symbols associated with paths in the trellis. In general [11] the generating function can be obtained as the transfer function of a state diagram regarded as a signal flow graph. The state diagram is defined over an expanded set of $N^2=2^{2\nu}$ states. For the special case of a trellis based upon a linear binary convolutional code, and a strongly homogeneous fair partition of a GGA, the minimum distance can be computed from a generating function obtained as the transfer function of a state diagram including only $N=2^{\nu}$ states. (See Theorem 3 of [15]).

We shall describe two examples of designs of four-dimensional Ungerboeck codes. The first example originates from Alphabet 2. It has minimum distance $2a^2=0.66$. The fair partition described before gives four subsets of 8 vectors each, with minimum intradistance $6a^2=2$. By choosing a 4-state trellis code with the structure described in Fig.6, we get a squared free distance $6a^2=2$. If this figure is compared to the minimum distance achieved by using two independent 4-PSK signals, which transmit the same amount of information over the same number of dimensions, we see that an energy saving of 3 dB is obtained.

Consider now Alphabet 4. It has a minimum square distance 0.3. The fair partition described gives 16 subalphabets of 8 vectors each, with minimum intradistance 1.2. By using the 4-state Ungerboeck code described in Fig.7, the squared free distance obtained is $d^2_{free}=1.2$. By comparing this to the minimum distance obtained by using two independent 8-PSK signals, we see that an energy saving of about 6 dB is obtained.

VII. CONCLUSIONS

In this paper we have introduced the concept of generalized group alphabets. The combination of these alphabets with block or trellis codes was also considered. Some actual designs show that consideration of GGA's may lead to transmission systems providing a good performance with bandlimited channels, at the price of a relatively modest complexity.

REFERENCES

[1] D.Slepian, "Bounds on Communication", Bell System Technical Journal, vol.42, May 1963, pp.681-707.

[2] S.G.Wilson, H.A.Sleeper, and N.K.Srinath, "Four-dimensional modulation and coding: An alternate to frequency reuse", Proceedings of ICC'84, Amsterdam, The Netherlands, May 1984, pp.919-923.

[3] S.G.Wilson and H.A.Sleeper, "Four-dimensional modulation and coding: An alternate to frequency reuse", Rept. No. UVA/528200/EE83/107, Communications Systems Laboratory, Dept. of Electrical Engineering, University of Virginia, Charlottesville, VA, September 1983.

[4] A.Gersho and V.Lawrence, "Multidimensional signal design for digital transmission over bandlimited channels", Proceedings of ICC'84, Amsterdam, The Netherlands, May 1984, pp.377-380.

[5] G.R.Welti and J.S.Lee, "Digital transmission with coherent four-dimensional modulation", IEEE Trans. on Inform. Theory, vol.IT-20, July 1974, pp.497-502.

[6] L.H.Zetterberg and H.Brandstrom, "Codes for combined phase

and amplitude modulated signals in a four-dimensional space", IEEE Trans.on Communications,vol.COM-25, Sept. 1977, pp.943-950.

[7] D.Slepian, "Group codes for the Gaussian channel", Bell System Technical Journal, vol.47, April 1968, pp.575-602.

[8] V.V.Ginzburg, "Mnogomerniye signaly dlya nepreryvnogo kanala", Problemy Peredaci Informacii, n.1, January-March 1984, pp.28-46, (in Russian). English translation: "Multidimensional signals for a continuous channel," Problems of Information Transmission, pp.20-34, 1984.

[9] F.J.MacWilliams and N.J.A.Sloane, "The Theory of Error-Correcting Codes", Amsterdam: North-Holland, 1977.

[10] G.Ungerboeck,"Channel coding with multilevel/phase signals" IEEE Trans. on Inform. Theory, vol.IT-28, January 1982, pp.55-67.

[11] E.Biglieri, "High-level modulation and coding for nonlinear satellite channels," IEEE Trans. on Communications, vol.COM-32, May 1984, pp.616-626.

[12] I.Jacobs, "Comparison of M-ary modulation systems", Bell System Technical Journal, vol.46, May-June 1967, pp.843-864

[13] D.Divsalar and J.H.Yuen, "Asymmetric MPSK for trellis codes", GLOBECOM '84, Atlanta, GA, November 26-29, 1984, pp.20.6.1-20.6.8.

[14] D.Divsalar and M.K.Simon, "Combined trellis coding with asymmetric modulations", Globecom '86, New Orleans, Louisiana, Dec.2-5, 1985, pp.21.2.1-21.2.7.

[15] E.Zehavi and J.K.Wolf, "On the performance evaluation of trellis codes", IEEE Trans. on Inform. Theory, to be published.

[16] E.Biglieri amd M.Elia, "On the existence of group codes for the Gaussian channel", IEEE Trans. on Inform. Theory, vol.IT-18, May 1972, pp.399-402.

[17] R.Calderbank and J.E.Mazo, "A new description of trellis codes", IEEE Trans. on Inform. Theory, vol.IT-30, November 1984, pp.784-791.

[18] H.Himai, S.Hirakawa, "A new multilevel coding method using error-correcting codes", IEEE Trans. on Inform. Theory, vol.IT-23, 1977, pp.371-377.

[19] J.H.Van Lint, "Introduction to Coding Theory", New York: Springer Verlag, 1982.

Figure captions

Fig. 1 Alphabet 2 and its fair partition

Fig. 2 Alphabet 3 and its fair partition

Fig. 3 Alphabet 4 and its fair partition

Fig. 4 Ginzburg construction

Fig. 5 An example of Ginzburg construction

Fig. 6 4-state Ungerboeck code for Alphabet 2

Fig. 7 4-state Ungerboeck code for Alphabet 4

$$
\begin{array}{cccc}
 & \mathrm{A} & & \\
a & a & a & 0 \\
0 & -a & a & a \\
a & 0 & -a & a \\
a & -a & 0 & -a \\
-a & -a & -a & 0 \\
0 & a & -a & -a \\
-a & 0 & a & -a \\
-a & a & 0 & a
\end{array}
\qquad
\begin{array}{cccc}
 & \mathrm{B} & & \\
a & a & 0 & a \\
-a & a & -a & 0 \\
0 & -a & -a & a \\
-a & 0 & a & a \\
-a & -a & 0 & -a \\
a & -a & a & 0 \\
0 & a & a & -a \\
a & 0 & -a & -a
\end{array}
\qquad
\begin{array}{cccc}
 & \mathrm{C} & & \\
a & 0 & a & a \\
a & a & 0 & -a \\
-a & a & a & 0 \\
0 & -a & a & -a \\
-a & 0 & -a & -a \\
-a & -a & 0 & a \\
a & -a & -a & 0 \\
0 & a & -a & a
\end{array}
\qquad
\begin{array}{cccc}
 & \mathrm{D} & & \\
0 & a & a & a \\
-a & 0 & -a & a \\
-a & a & 0 & -a \\
a & a & -a & 0 \\
0 & -a & -a & -a \\
a & 0 & a & -a \\
a & -a & 0 & a \\
-a & -a & a & 0
\end{array}
$$

Fig. 1

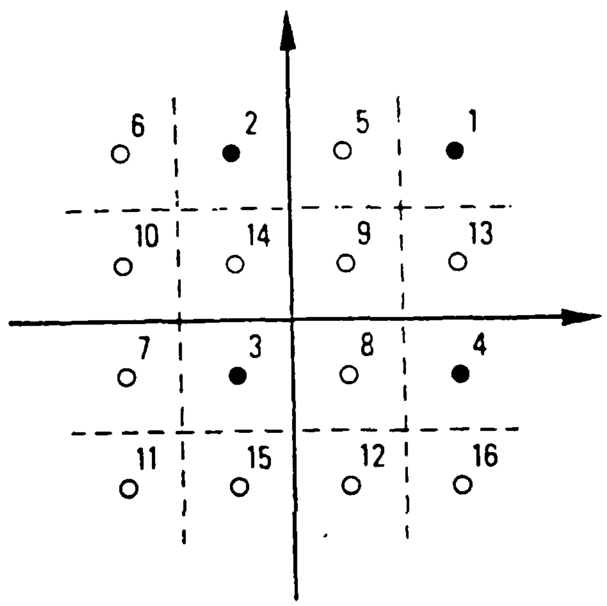

Fig. 2

A	B	C	D	E	F	G	H
c c c 0	c c 0 c	c 0 c c	0 c c c	-c c c 0	c c 0-c	c 0-c c	0-c c c
-b c c 0	c c 0-b	c 0-b c	0-b c c	b c c 0	c c 0 b	c 0 b c	0 b c c
c-b c 0	-b c 0 c	c 0 c-b	0 c-b c	-c-b c 0	-b c 0-c	c 0-c-b	0-c-b c
c c-b 0	c-b 0 c	-b 0 c c	0 c c b	-c c-b 0	c-b 0-c	-b 0-c c	0-c c-b
-c-c-c 0	-c-c 0-c	-c 0-c-c	0-c-c-c	c-c-c 0	-c-c 0 c	-c 0 c-c	0 c-c-c
b-c-c 0	-c-c 0 b	-c 0 b-c	0 b-c-c	-b-c-c 0	-c-c 0-b	-c 0-b-c	0-b-c-c
-c b-c 0	b-c 0-c	-c 0-c b	0-c b-c	c b c 0	b-c 0 c	-c 0 c b	0 c b-c
-c-c b 0	-c b 0-c	b 0-c-c	0-c-c b	c-c b 0	-c b 0 c	b 0 c-c	0 c-c b

I	J	K	L	M	N	O	P
c-c c 0	-c c 0 c	c 0 c-c	0 c-c c	c c-c 0	c-c 0 c	-c 0 c c	0 c c-c
-b-c c 0	-c c 0-b	c 0-b-c	0-b-c c	-b c-c 0	c-c 0-b	-c 0-b c	0-b c-c
c b c 0	b c 0 c	c 0 c b	0 c b c	c-b c 0	-b-c 0 c	-c 0 c-b	0 c-b-c
c-c-b 0	-c-b 0 c	-b 0 c-c	0 c-c-b	c c b 0	c b 0 c	b 0 c c	0 c c b
-c c-c 0	c-c 0-c	-c 0-c c	0-c c-c	-c-c c 0	-c c 0-c	c 0-c-c	0-c-c c
b c-c 0	c-c 0 b	-c 0 b c	0 b c-c	b-c c 0	-c c 0 b	c 0 b-c	0 b-c c
-c-b-c 0	-b-c 0-c	-c 0-c-b	0-c-b-c	-c b c 0	b c 0-c	c 0-c b	0-c b c
-c c b 0	c b 0-c	b 0-c c	0-c c b	-c-b-c 0	-c-b 0-c	-b 0-c-c	0-c-c-b

Fig. 3

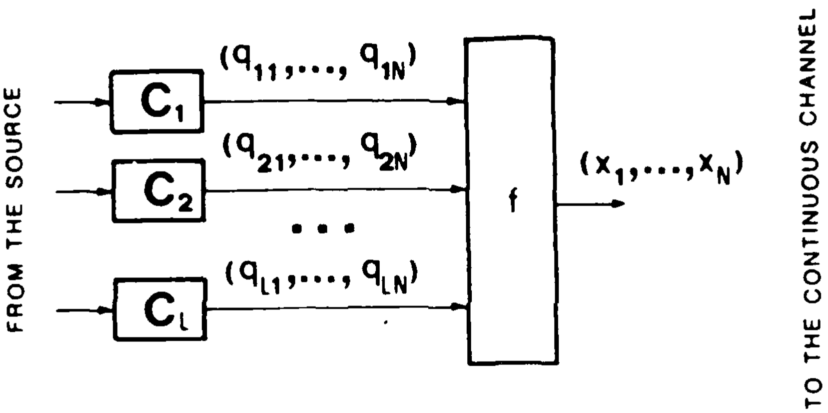
FROM THE SOURCE
C_1
C_2
C_L
$(q_{11},\dots,q_{1N})$
$(q_{21},\dots,q_{2N})$
$(q_{L1},\dots,q_{LN})$
f
$(x_1,\dots,x_N)$
TO THE CONTINUOUS CHANNEL

Fig. 4

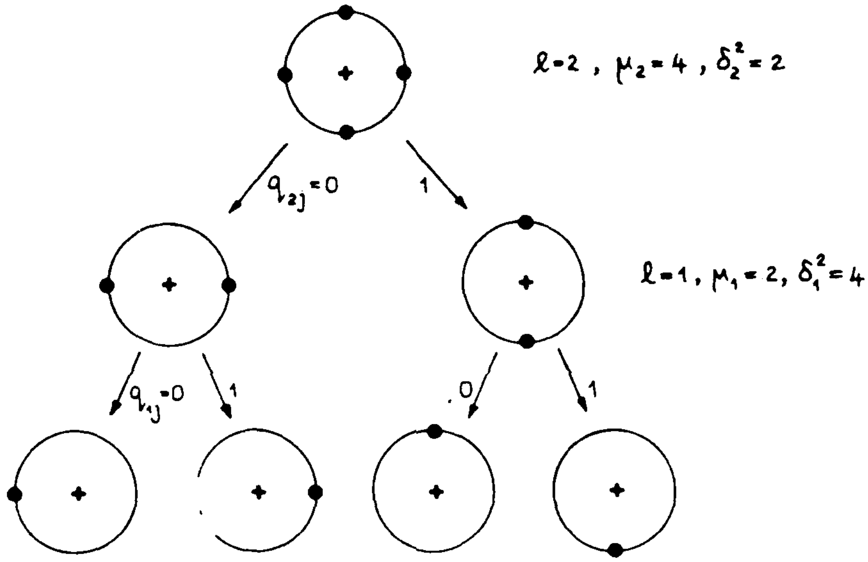

Fig. 5

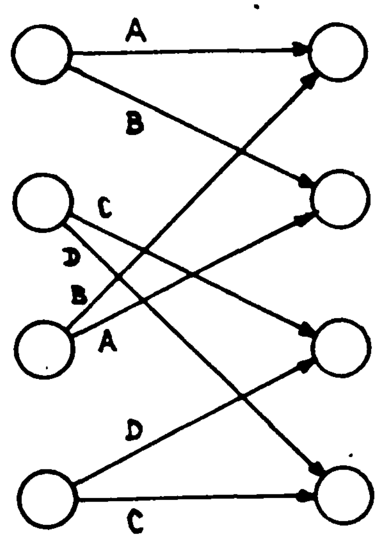

Fig. 6

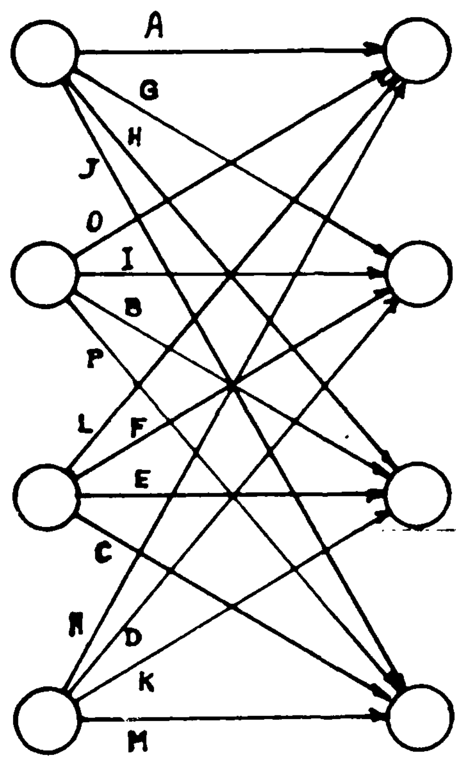

Fig. 7

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